

# Accurate Design of Optical Microring Resonators

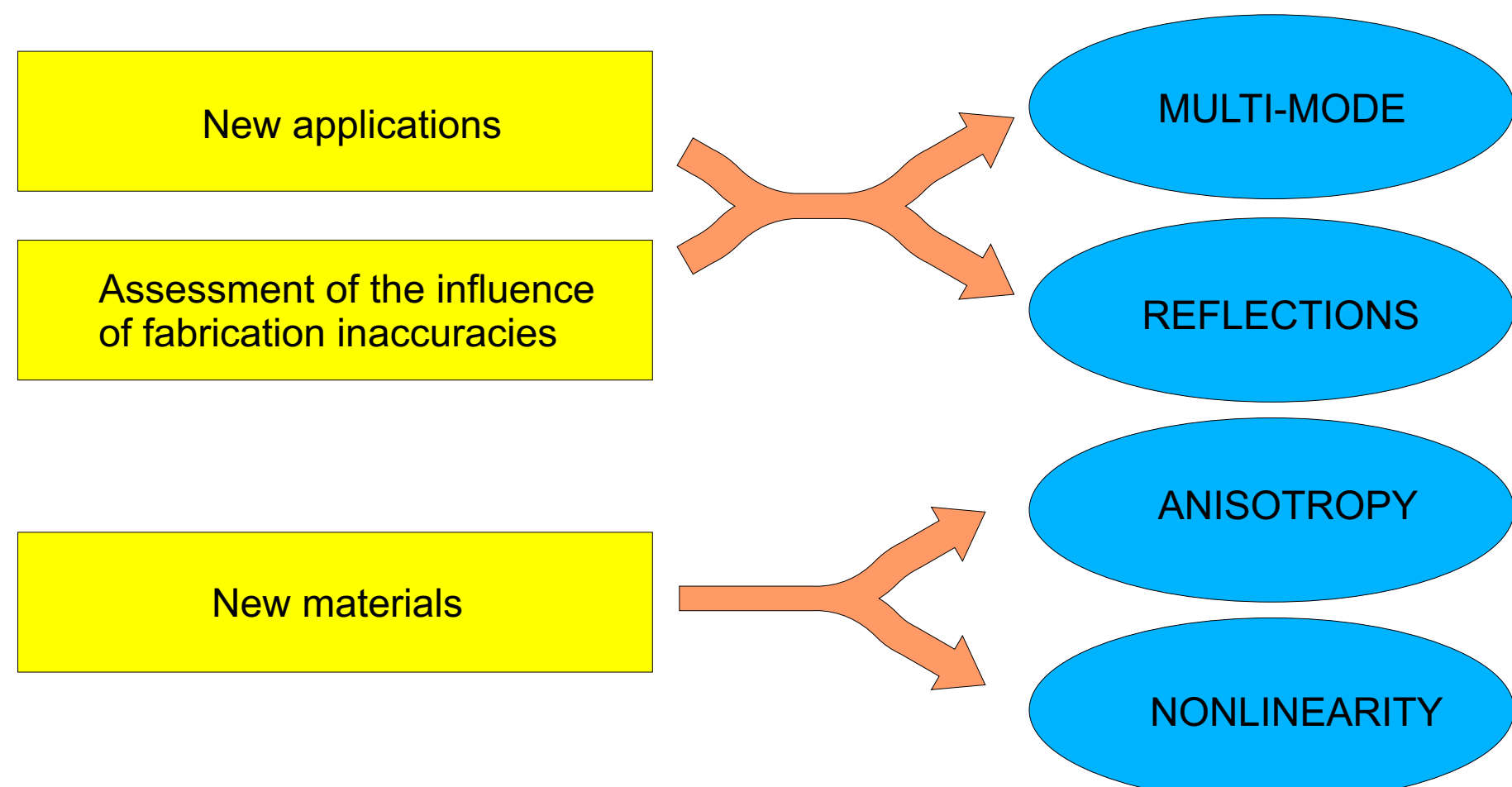
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## Range of applications for Microring Resonators (MRR)

Modulators  
Polarization splitter  
Multiplexing / demultiplexing

Sensors  
Photodetectors  
Optical trimming  
Plasmonics (nano-rice chain)

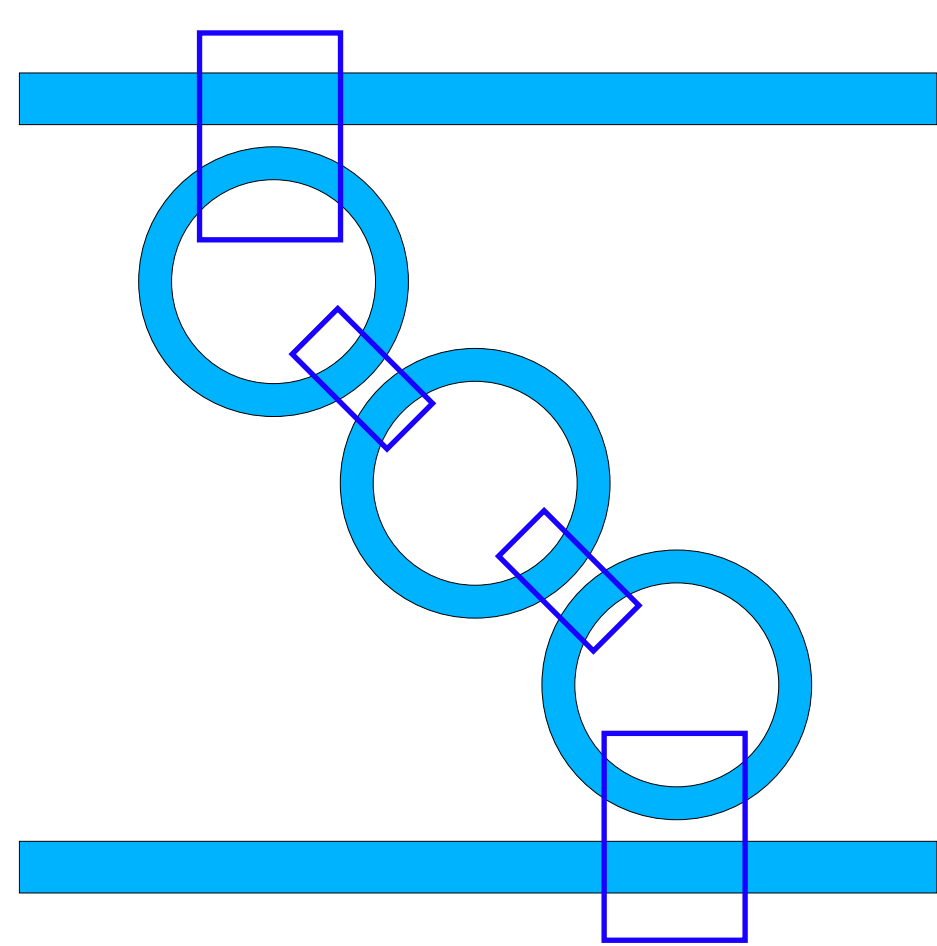
## New challenges for the MRR models



## Comparison of commonly used methodologies

Methodology	MRR Radius	Multi-mode	Reflections	Anisotropy	Nonlinearity
FDTD	< 30um	✓	✓	✓	✓
Mode Solver + CMT	> 30um	✗	✗	possible	✗
BPM + Transfer Matrix	> 30um	✓	✗	possible	✓
EME + Bend Solver + Transfer Matrices (our methodology)	> 30um	✓	✓	✓	✗

## The EME-based methodology

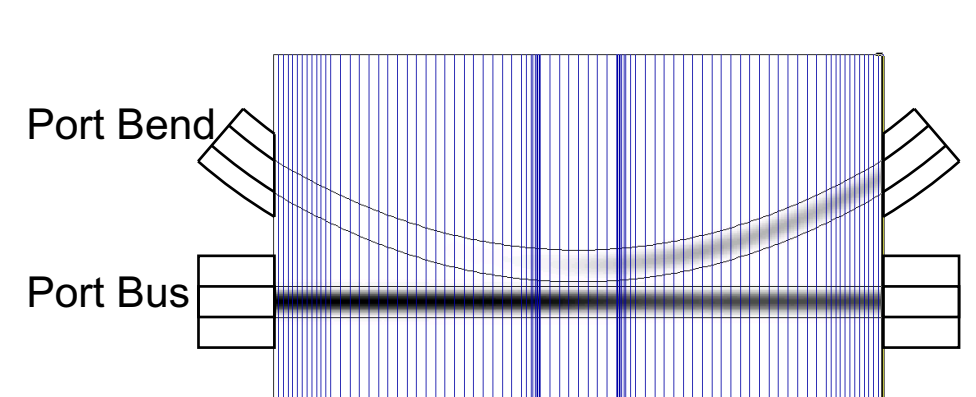


Step 1: Modelling of the coupling region by EigenMode Expansion (EME) method using FIMMPROP [1-3]

Step 2: Modelling of the propagation and loss in the rings by using the FIMMWAIVE Bend Mode Solver (finite-difference FDM, finite-element FEM, film-mode matching FMM [3,6])

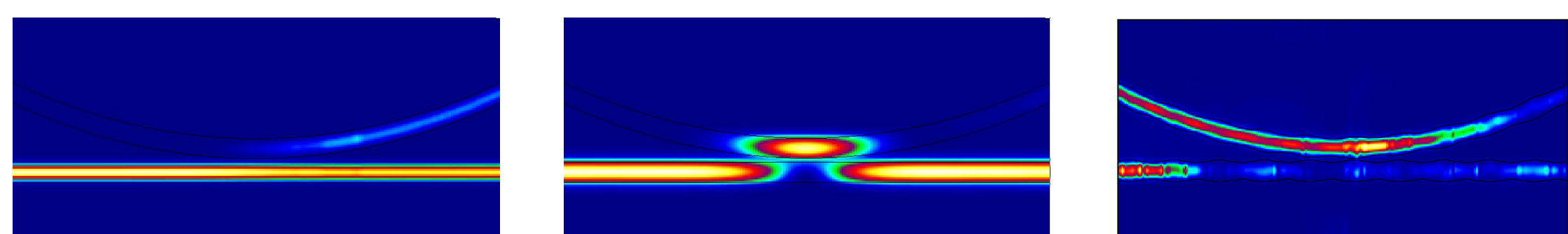
Step 3: Calculation of the device response using Transfer Matrix approach

## Step 1: Modelling of the coupling region with EME

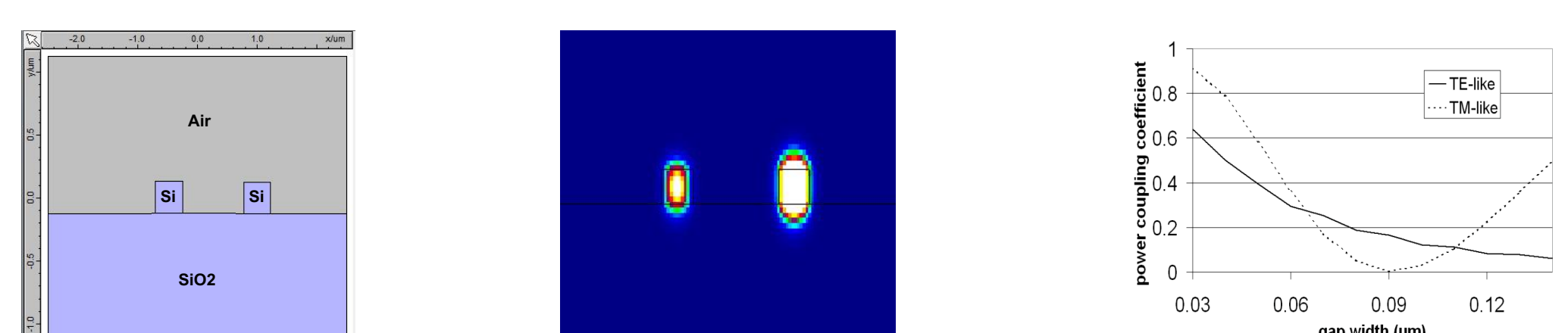


Implementation of EME in FIMMPROP in the case of a ring coupler:  
- Calculation of a set of local XY modes  
- Taper algorithm: effective Z-discretisation  
- Calculation of the Scattering Matrix for the coupler based on mode overlap integrals  
- Addition of Input / Output Ports (coupling to bend modes in the ring and straight modes in the bus waveguide)

### Field intensity in the SOI coupler [4, 7]



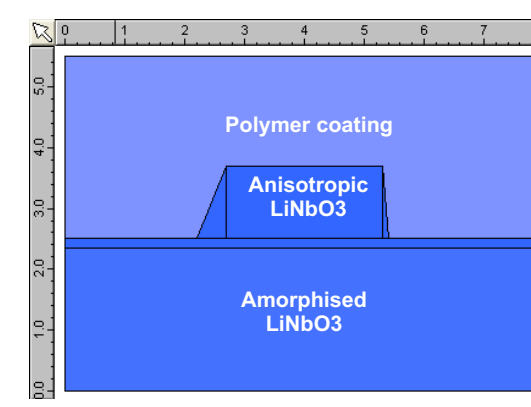
### Field intensity in XY cross-section



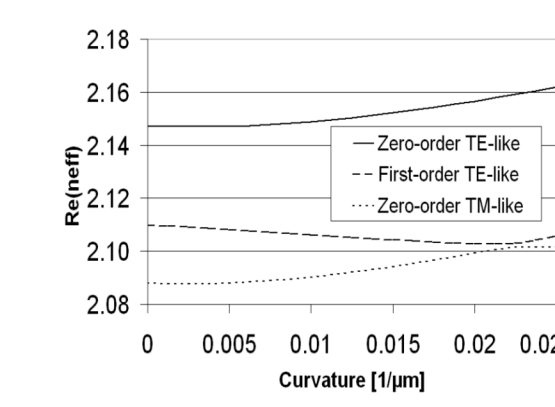
SOI parameters: ring radius 40 um, gap width 90 nm, wavelength 1.55 um

## Step 2: Bend mode calculation

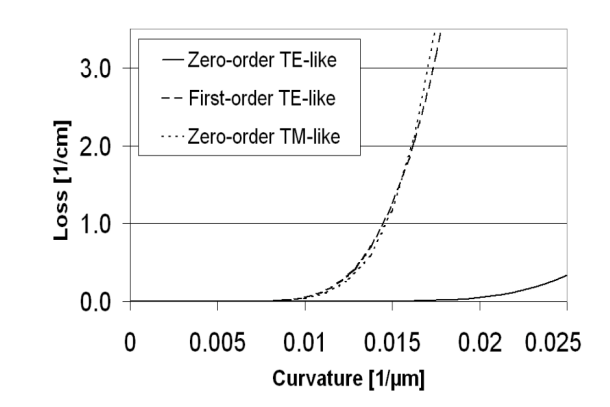
Fully vectorial FIMMWAIVE Bend Mode Solvers:  
- calculation with the FMM Solver in cylindrical coordinates [3]  
- with the FDM and FEM in "local cylindrical coordinates" [6]



XY cross-section

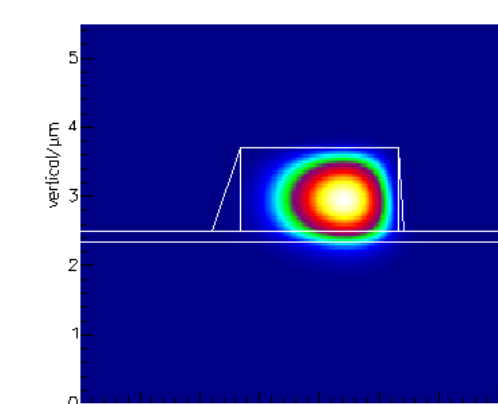


effective index for the bend modes v. curvature

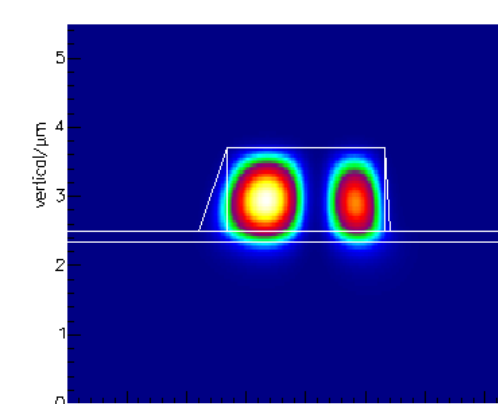


bend mode loss v. curvature

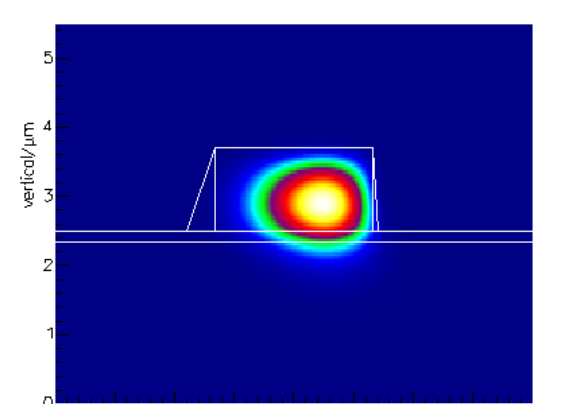
Bend modes for LiNbO3 MRR (radius 80um) [5] calculated with FDM Solver:



fundamental TE-like



fundamental TM-like

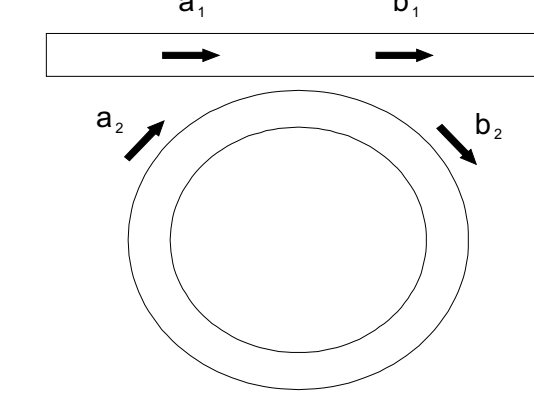


first-order TE-like

## Step 3: Transfer Matrix calculation

Transfer Matrix derivation for the case of a single-ring multi-mode resonator with reflections:

Using the scattering matrices for the coupling region with ports calculated by EME method (step 1):



$$\begin{aligned} \begin{bmatrix} \bar{a}_{1out} \\ \bar{a}_{2out} \end{bmatrix} &= \begin{bmatrix} T_{11}^L & T_{12}^L \\ T_{21}^L & T_{22}^L \end{bmatrix} \begin{bmatrix} \bar{a}_{1in} \\ \bar{a}_{2in} \end{bmatrix} + \begin{bmatrix} R_{11}^L & R_{12}^L \\ R_{21}^L & R_{22}^L \end{bmatrix} \begin{bmatrix} \bar{b}_{1in} \\ \bar{b}_{2in} \end{bmatrix} \quad [1] \\ \begin{bmatrix} \bar{b}_{1out} \\ \bar{b}_{2out} \end{bmatrix} &= \begin{bmatrix} T_{11}^R & T_{12}^R \\ T_{21}^R & T_{22}^R \end{bmatrix} \begin{bmatrix} \bar{a}_{1in} \\ \bar{a}_{2in} \end{bmatrix} + \begin{bmatrix} R_{11}^R & R_{12}^R \\ R_{21}^R & R_{22}^R \end{bmatrix} \begin{bmatrix} \bar{b}_{1in} \\ \bar{b}_{2in} \end{bmatrix} \quad [2] \\ \begin{bmatrix} \bar{a}_{1in} \\ \bar{a}_{2in} \end{bmatrix} &= \begin{bmatrix} T_{11}^L & T_{12}^L \\ T_{21}^L & T_{22}^L \end{bmatrix} \begin{bmatrix} \bar{a}_{1out} \\ \bar{a}_{2out} \end{bmatrix} + \begin{bmatrix} R_{11}^L & R_{12}^L \\ R_{21}^L & R_{22}^L \end{bmatrix} \begin{bmatrix} \bar{b}_{1in} \\ \bar{b}_{2in} \end{bmatrix} \quad [3] \\ \begin{bmatrix} \bar{b}_{1in} \\ \bar{b}_{2in} \end{bmatrix} &= \begin{bmatrix} T_{11}^R & T_{12}^R \\ T_{21}^R & T_{22}^R \end{bmatrix} \begin{bmatrix} \bar{a}_{1in} \\ \bar{a}_{2in} \end{bmatrix} + \begin{bmatrix} R_{11}^R & R_{12}^R \\ R_{21}^R & R_{22}^R \end{bmatrix} \begin{bmatrix} \bar{b}_{1out} \\ \bar{b}_{2out} \end{bmatrix} \quad [4] \end{aligned}$$

where  $T_{ij}^{L(R)}$  and  $R_{ij}^{L(R)}$  denote transmittance ( $T$ ) and reflectance ( $R$ ) scattering matrices from left hand side ( $L$ ) or from right hand side ( $R$ ) from port  $i$  to  $j$ .  $\bar{a}(b)_{in(out)}$  denotes the vector of the mode amplitudes of the  $i^{th}$  port  $a_i(b_i)$ ,  $in/out$  means coming in or out of the corresponding port.

Using the results of the bend mode calculations in the step 2, we can write the scattering matrix for the propagation in the arc of the ring as:

$$\begin{aligned} \begin{bmatrix} \bar{a}_{2in} \\ \bar{b}_{2in} \end{bmatrix} &= \begin{bmatrix} M_{ring} & 0 \\ 0 & e^{i\beta L_{ring}} \end{bmatrix} \begin{bmatrix} \bar{a}_{1in} \\ \bar{b}_{1in} \end{bmatrix} \quad [5] \\ \begin{bmatrix} \bar{b}_{2out} \\ \bar{a}_{2out} \end{bmatrix} &= \begin{bmatrix} M_{ring} & 0 \\ 0 & e^{i\beta L_{ring}} \end{bmatrix} \begin{bmatrix} \bar{b}_{1out} \\ \bar{a}_{1out} \end{bmatrix} \quad [6] \end{aligned}$$

where  $\beta$  is the complex propagation constant of the  $i^{th}$  mode in the bent waveguides,  $n$  is the number of modes considered in the bend,  $L_{ring}$  is the length of the arc of the ring.

From the equations (1-6) we can derive:

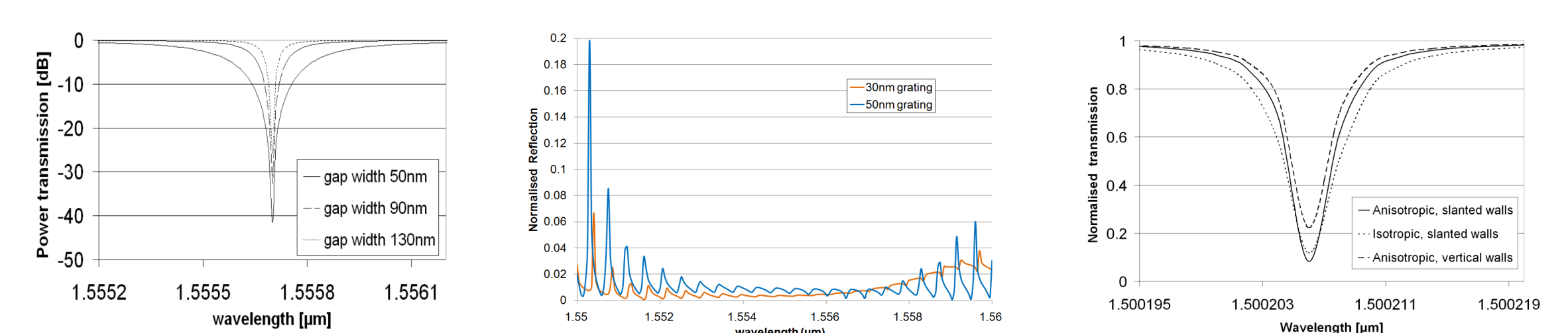
$$\begin{bmatrix} [I] - [T_{22}^L] M_{ring} & -[R_{22}^R] M_{ring} \\ -[R_{21}^L] M_{ring} & [I] - [T_{22}^R] M_{ring} \end{bmatrix} \begin{bmatrix} \bar{b}_{2out} \\ \bar{a}_{2out} \end{bmatrix} = \begin{bmatrix} T_{21}^L & R_{21}^R \\ R_{21}^L & T_{21}^R \end{bmatrix} \begin{bmatrix} \bar{a}_{1in} \\ \bar{b}_{1in} \end{bmatrix} \quad [7]$$

$$\begin{bmatrix} \bar{b}_{1out} \\ \bar{a}_{1out} \end{bmatrix} = \begin{bmatrix} T_{11}^L & T_{12}^L \\ T_{21}^L & T_{22}^L \end{bmatrix} \begin{bmatrix} \bar{a}_{1in} \\ \bar{a}_{2in} \end{bmatrix} + \begin{bmatrix} R_{11}^L & R_{12}^L \\ R_{21}^L & R_{22}^L \end{bmatrix} \begin{bmatrix} \bar{b}_{1in} \\ \bar{b}_{2in} \end{bmatrix} \quad [8]$$

$$\begin{bmatrix} \bar{a}_{1in} \\ \bar{a}_{2in} \end{bmatrix} = \begin{bmatrix} T_{11}^R & T_{12}^R \\ T_{21}^R & T_{22}^R \end{bmatrix} \begin{bmatrix} \bar{a}_{1out} \\ \bar{a}_{2out} \end{bmatrix} + \begin{bmatrix} R_{11}^R & R_{12}^R \\ R_{21}^R & R_{22}^R \end{bmatrix} \begin{bmatrix} \bar{b}_{1in} \\ \bar{b}_{2in} \end{bmatrix} \quad [9]$$

where  $[I]$  is the identity matrix. Solving the last equations we obtain the mode amplitudes  $\bar{a}_{1out}$ ,  $\bar{b}_{1out}$ .

## Transmission and Reflection of Microring Resonators



(left) Transmission spectrum around resonance for the TE-like polarisation, plotted in dB for a gap width of 50nm, 90nm and 130nm (SOI resonator [5]).  
(middle) Reflection spectrum of the SOI resonator with waveguide widths modulated by sinusoidal gratings. The grating amplitudes are equivalent to typical surface roughnesses due to fabrication of 30 nm (orange line) and 50 nm (blue line) [7]. The period of the grating is 1 um.  
(right) Transmission of the original fluorine-implanted LiNbO3 [4] microring resonator (solid line), of the same resonator with vertical wall waveguides (dashed line) and of the same waveguide with isotropic refractive index (dotted line).

## Conclusions

- 1) We introduced a Micro Ring Resonator model based on:  
\* fully vectorial modelling of the coupling region (accounting for anisotropic materials and the reflections);  
\* multimode (bend mode) propagation in the arc of the ring;  
\* the transfer matrix approach to combine together the regions of the ring resonator model.
- 2) The methodology was tested on the modelling of a single SOI microring resonator comparing the calculated results with published experimental results [4];
- 3) We implemented the methodology for two different cases of microring resonators (SOI and LiNbO3 based);
- 4) On the example of LiNbO3 micro-ring resonators, we demonstrated the influence of material anisotropy and complex cross-section geometry on the MRR transmission spectrum.
- 5) On the example of SOI micro-ring resonator, we assessed the influence of surface roughness on the reflection spectra.
- 6) Preliminary calculations of steps 1 and 2 can assist in optimising the MRR parameters. For example, we demonstrated that the bend mode calculations can help in choosing the range of microring radii for which single-mode operation can be achieved in the ring.

## References

- [1] D.F.G. Gallagher, T.P. Felici, Proc. SPIE, vol. 4987, p. 69 (2003).
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